# **1.4** | Total Internal Reflection

## **Learning Objectives**

By the end of this section, you will be able to:

- Explain the phenomenon of total internal reflection
- Describe the workings and uses of optical fibers
- Analyze the reason for the sparkle of diamonds

A good-quality mirror may reflect more than 90% of the light that falls on it, absorbing the rest. But it would be useful to have a mirror that reflects all of the light that falls on it. Interestingly, we can produce total reflection using an aspect of refraction.

Consider what happens when a ray of light strikes the surface between two materials, as shown in **Figure 1.14**(a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than for the first, the ray bends away from the perpendicular. (Since  $n_1 > n_2$ , the angle of refraction is greater than the angle of incidence—that is,  $\theta_2 > \theta_1$ .) Now imagine what happens as the incident angle increases. This causes  $\theta_2$  to increase also. The largest the angle of refraction  $\theta_2$  can be is 90°, as shown in part (b). The **critical angle**  $\theta_c$  for a combination of materials is defined to be the incident angle  $\theta_1$  that produces an angle of refraction of 90°. That is,  $\theta_c$  is the incident angle for which  $\theta_2 = 90^\circ$ . If the incident angle  $\theta_1$  is greater than the critical angle, as shown in **Figure 1.14**(c), then all of the light is reflected back into medium 1, a condition called **total internal reflection**. (As the figure shows, the reflected rays obey the law of reflection so that the angle of reflection is equal to the angle of incidence in all three cases.)



**Figure 1.14** (a) A ray of light crosses a boundary where the index of refraction decreases. That is,  $n_2 < n_1$ . The ray bends away from the perpendicular. (b) The critical angle  $\theta_c$  is the angle of incidence for which the angle of refraction is 90°. (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Snell's law states the relationship between angles and indices of refraction. It is given by

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

When the incident angle equals the critical angle ( $\theta_1 = \theta_c$ ), the angle of refraction is 90° ( $\theta_2 = 90^\circ$ ). Noting that  $\sin 90^\circ = 1$ , Snell's law in this case becomes

$$n_1 \sin \theta_1 = n_2$$

The critical angle  $\theta_c$  for a given combination of materials is thus

(1.5)

$$\theta_{\rm c} = \sin^{-1} \left( \frac{n_2}{n_1} \right) \text{ for } n_1 > n_2.$$

Total internal reflection occurs for any incident angle greater than the critical angle  $\theta_c$ , and it can only occur when the second medium has an index of refraction less than the first. Note that this equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in **Figure 1.14**.

### Example 1.4

#### **Determining a Critical Angle**

What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air? The index of refraction for polystyrene is 1.49.

#### Strategy

The index of refraction of air can be taken to be 1.00, as before. Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and we can use the equation

$$\theta_{\rm c} = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

to find the critical angle  $\theta_c$ , where  $n_2 = 1.00$  and  $n_1 = 1.49$ .

#### Solution

Substituting the identified values gives

$$\theta_{\rm c} = \sin^{-1}\left(\frac{1.00}{1.49}\right) = \sin^{-1}(0.671) = 42.2^{\circ}$$

#### Significance

This result means that any ray of light inside the plastic that strikes the surface at an angle greater than  $42.2^{\circ}$  is totally reflected. This makes the inside surface of the clear plastic a perfect mirror for such rays, without any need for the silvering used on common mirrors. Different combinations of materials have different critical angles, but any combination with  $n_1 > n_2$  can produce total internal reflection. The same calculation as made here shows

that the critical angle for a ray going from water to air is  $48.6^{\circ}$ , whereas that from diamond to air is  $24.4^{\circ}$ , and that from flint glass to crown glass is  $66.3^{\circ}$ .



**1.3** Check Your Understanding At the surface between air and water, light rays can go from air to water and from water to air. For which ray is there no possibility of total internal reflection?

In the photo that opens this chapter, the image of a swimmer underwater is captured by a camera that is also underwater. The swimmer in the upper half of the photograph, apparently facing upward, is, in fact, a reflected image of the swimmer below. The circular ripple near the photograph's center is actually on the water surface. The undisturbed water surrounding it makes a good reflecting surface when viewed from below, thanks to total internal reflection. However, at the very top edge of this photograph, rays from below strike the surface with incident angles less than the critical angle, allowing the camera to capture a view of activities on the pool deck above water.

## Fiber Optics: Endoscopes to Telephones

Fiber optics is one application of total internal reflection that is in wide use. In communications, it is used to transmit telephone, internet, and cable TV signals. **Fiber optics** employs the transmission of light down fibers of plastic or glass. Because the fibers are thin, light entering one is likely to strike the inside surface at an angle greater than the critical angle and, thus, be totally reflected (**Figure 1.15**). The index of refraction outside the fiber must be smaller than inside. In fact, most fibers have a varying refractive index to allow more light to be guided along the fiber through total internal refraction. Rays are reflected around corners as shown, making the fibers into tiny light pipes.



Bundles of fibers can be used to transmit an image without a lens, as illustrated in **Figure 1.16**. The output of a device called an endoscope is shown in **Figure 1.16**(b). Endoscopes are used to explore the interior of the body through its natural orifices or minor incisions. Light is transmitted down one fiber bundle to illuminate internal parts, and the reflected light is transmitted back out through another bundle to be observed.



**Figure 1.16** (a) An image "A" is transmitted by a bundle of optical fibers. (b) An endoscope is used to probe the body, both transmitting light to the interior and returning an image such as the one shown of a human epiglottis (a structure at the base of the tongue). (credit b: modification of work by "Med\_Chaos"/Wikimedia Commons)

Fiber optics has revolutionized surgical techniques and observations within the body, with a host of medical diagnostic and therapeutic uses. Surgery can be performed, such as arthroscopic surgery on a knee or shoulder joint, employing cutting tools attached to and observed with the endoscope. Samples can also be obtained, such as by lassoing an intestinal polyp for external examination. The flexibility of the fiber optic bundle allows doctors to navigate it around small and difficult-to-reach regions in the body, such as the intestines, the heart, blood vessels, and joints. Transmission of an intense laser beam to burn away obstructing plaques in major arteries, as well as delivering light to activate chemotherapy drugs, are becoming commonplace. Optical fibers have in fact enabled microsurgery and remote surgery where the incisions are small and the

surgeon's fingers do not need to touch the diseased tissue.

Optical fibers in bundles are surrounded by a cladding material that has a lower index of refraction than the core (**Figure 1.17**). The cladding prevents light from being transmitted between fibers in a bundle. Without cladding, light could pass between fibers in contact, since their indices of refraction are identical. Since no light gets into the cladding (there is total internal reflection back into the core), none can be transmitted between clad fibers that are in contact with one another. Instead, the light is propagated along the length of the fiber, minimizing the loss of signal and ensuring that a quality image is formed at the other end. The cladding and an additional protective layer make optical fibers durable as well as flexible.



lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another.

Special tiny lenses that can be attached to the ends of bundles of fibers have been designed and fabricated. Light emerging from a fiber bundle can be focused through such a lens, imaging a tiny spot. In some cases, the spot can be scanned, allowing quality imaging of a region inside the body. Special minute optical filters inserted at the end of the fiber bundle have the capacity to image the interior of organs located tens of microns below the surface without cutting the surface—an area known as nonintrusive diagnostics. This is particularly useful for determining the extent of cancers in the stomach and bowel.

In another type of application, optical fibers are commonly used to carry signals for telephone conversations and internet communications. Extensive optical fiber cables have been placed on the ocean floor and underground to enable optical communications. Optical fiber communication systems offer several advantages over electrical (copper)-based systems, particularly for long distances. The fibers can be made so transparent that light can travel many kilometers before it becomes dim enough to require amplification—much superior to copper conductors. This property of optical fibers is called low loss. Lasers emit light with characteristics that allow far more conversations in one fiber than are possible with electric signals on a single conductor. This property of optical fibers is called high bandwidth. Optical signals in one fiber do not produce undesirable effects in other adjacent fibers. This property of optical fibers is called reduced crosstalk. We shall explore the unique characteristics of laser radiation in a later chapter.

### **Corner Reflectors and Diamonds**

Corner reflectors (**The Law of Reflection**) are perfectly efficient when the conditions for total internal reflection are satisfied. With common materials, it is easy to obtain a critical angle that is less than 45°. One use of these perfect mirrors is in binoculars, as shown in **Figure 1.18**. Another use is in periscopes found in submarines.



(prisms) with total internal reflection to get light to the observer's eyes.

Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only  $24.4^{\circ}$ , so when light enters a diamond, it has trouble getting back out (Figure 1.19). Although light freely enters the diamond, it can exit only if it makes an angle less than  $24.4^{\circ}$ . Facets on diamonds are specifically intended to make this unlikely. Good diamonds are very clear, so that the light makes many internal reflections and is concentrated before exiting—hence the bright sparkle. (Zircon is a natural gemstone that has an exceptionally large index of refraction, but it is not as large as diamond, so it is not as highly prized. Cubic zirconia is manufactured and has an even higher index of refraction ( $\approx 2.17$ ), but it is still less than that of diamond.) The colors

you see emerging from a clear diamond are not due to the diamond's color, which is usually nearly colorless. The colors result from dispersion, which we discuss in **Dispersion**. Colored diamonds get their color from structural defects of the crystal lattice and the inclusion of minute quantities of graphite and other materials. The Argyle Mine in Western Australia produces around 90% of the world's pink, red, champagne, and cognac diamonds, whereas around 50% of the world's clear diamonds come from central and southern Africa.



**Figure 1.19** Light cannot easily escape a diamond, because its critical angle with air is so small. Most reflections are total, and the facets are placed so that light can exit only in particular ways—thus concentrating the light and making the diamond sparkle brightly.

Explore **refraction and reflection of light (https://openstaxcollege.org/l/21bendoflight)** between two media with different indices of refraction. Try to make the refracted ray disappear with total internal reflection. Use the protractor tool to measure the critical angle and compare with the prediction from **Equation 1.5**.

# **1.5** Dispersion

## **Learning Objectives**

By the end of this section, you will be able to:

- Explain the cause of dispersion in a prism
- Describe the effects of dispersion in producing rainbows
- · Summarize the advantages and disadvantages of dispersion

Everyone enjoys the spectacle of a rainbow glimmering against a dark stormy sky. How does sunlight falling on clear drops of rain get broken into the rainbow of colors we see? The same process causes white light to be broken into colors by a clear glass prism or a diamond (Figure 1.20).





(a)

(b)

**Figure 1.20** The colors of the rainbow (a) and those produced by a prism (b) are identical. (credit a: modification of work by "Alfredo55"/Wikimedia Commons; credit b: modification of work by NASA)

We see about six colors in a rainbow—red, orange, yellow, green, blue, and violet; sometimes indigo is listed, too. These colors are associated with different wavelengths of light, as shown in **Figure 1.21**. When our eye receives pure-wavelength light, we tend to see only one of the six colors, depending on wavelength. The thousands of other hues we can sense in other situations are our eye's response to various mixtures of wavelengths. White light, in particular, is a fairly uniform mixture of all visible wavelengths. Sunlight, considered to be white, actually appears to be a bit yellow, because of its mixture of wavelengths, but it does contain all visible wavelengths. The sequence of colors in rainbows is the same sequence as the colors shown in the figure. This implies that white light is spread out in a rainbow according to wavelength. **Dispersion** is defined as the spreading of white light into its full spectrum of wavelengths. More technically, dispersion occurs whenever the propagation of light depends on wavelength.



distribution of colors according to wavelengths.

Any type of wave can exhibit dispersion. For example, sound waves, all types of electromagnetic waves, and water waves can be dispersed according to wavelength. Dispersion may require special circumstances and can result in spectacular displays such as in the production of a rainbow. This is also true for sound, since all frequencies ordinarily travel at the same speed. If you listen to sound through a long tube, such as a vacuum cleaner hose, you can easily hear it dispersed by